Pearson Edexcel

Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International A Level
In Pure Mathematics (WMA11) Paper 01

General

This paper proved to be a fair paper on the WMA11 content, and it was pleasing to see candidates were able to make attempts at all the questions. Overall, marks were available to candidates of all abilities and the questions which proved to be most challenging were 8 and 10. Time did not appear to be an issue for candidates.

In general, there seemed to be far more candidates attempting answers next to the questions. This caused problems for some who would miscopy when they tried to transfer part of their solutions to other areas. Candidates should, therefore, be encouraged to write their solutions in the designated areas so that they can show all the necessary detail and working.

Candidates should also be reminded to pay particular attention to any information at the start of questions in emboldened writing, which may indicate the degree to which a calculator can be used and any further emphasis on showing all stages of their working, in order to score all the available marks.

Report on individual questions

Question 1

This proved to be a well attempted, accessible question and made a positive introduction to the paper. The question required candidates to find the first and second derivative of a three-term expression with positive and negative indices and many candidates achieved full marks. Integrating the expression rather than differentiating was seen on occasion and no marks were available to candidates who made this error.

Part (a) required candidates to find the first derivative and was particularly well attempted with most achieving the first two marks. The most common errors occurred when differentiating the second term, with some candidates rewriting the term as $3x^2$ rather than $3x^{-2}$, and others making an error when subtracting 1 from the power giving the derivative of $6x^{-2}$ as $6x^{-1}$; sign errors in the coefficients were also seen. Some candidates were unsure how to deal with the term in *x* and this was sometimes left unchanged or disappeared completely. Multiplying by the new index rather than the original one e.g. $5x^3$ differentiated to $10x^2$ was a less common error but was seen on occasion.

Part (b) instructed candidates to find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. It was clear that some candidates were unfamiliar with this notation and did not understand what was required of them. This included many candidates who had demonstrated proficiency with the process of differentiation in part (a). While some chose to miss this part out, there were several common incorrect attempts seen: integrating the original expression, integrating their answer from part (a), squaring each term of their answer to part (a), and squaring their entire answer to part (a). Where candidates understood the demand of the question, the vast majority were able to differentiate successfully and scored full marks. If marks were lost, it was often due to following through an incorrect answer from part (a) or making a sign error when differentiating the term with the negative index.

Overall, this question offered five routine marks, of which most candidates were able to score most of them.

Question 2

This question on indices was more challenging than expected for candidates with it being less common for full marks to be scored.

In part (a), most candidates correctly achieved the required answered and many gave their answer as $\frac{1}{8}x$, although some gave their answer as $\frac{1}{8}x^1$ or $\frac{x}{8}$. A small number of candidates incorrectly gave their answer as $\pm\frac{1}{8}x$, hence scoring B0. A few mistakenly wrote $\frac{1}{8x}$.

A large number of candidates correctly answered part (b), with most giving their answer as either $\frac{1}{256}x^{\frac{3}{2}}$ or $\frac{x^{\frac{3}{2}}}{256}$. Some candidates left their final answer as $\frac{1}{256x^{-\frac{3}{2}}}$ or $\frac{1}{256}x\sqrt{x}$ which, although equivalent, did not satisfy the required form stated in the question. The most common incorrect answer achieved was $\frac{1}{256}x^{-\frac{3}{2}}$. A small number of candidates incorrectly rewrote $\frac{1}{256x^{-\frac{3}{2}}}$ as $\frac{1}{256}x^{\frac{2}{3}}$ or $256x^{\frac{3}{2}}$.

Part (c) was found to be particularly challenging by candidates and many were unable to work their way through the problem by breaking it down and applying the rules of indices. A small number of candidates left their final answer as $\frac{16}{x^2}$ and so did not meet the required form given in the question, scoring M1A0. In the vast majority of successful responses, candidates simplified the inside of the bracket $\frac{ab}{2}$ first which led them to achieve $\left(\frac{1}{8}x^{\frac{3}{2}}\right)^{-\frac{4}{3}}$. Whilst most were then successful in applying the power of $-\frac{4}{3}$, a small number of candidates did have difficulty processing the power of x, and of these, most added the powers leading to $\ldots x^{\frac{1}{6}}$, so usually $(16x)^{\frac{1}{6}}$. When candidates did not simplify $\frac{ab}{2}$ first, they were usually unsuccessful in applying the power of $-\frac{4}{3}$ correctly. A small number managed to apply indices to the numbers correctly to get 16.

Question 3

This question on surds stated that "you must show all stages of your working" and "solutions relying on calculator technology are not acceptable." Most marks were lost where such technology was used or there was insufficient working shown to demonstrate that their solution did not require the use of it.

In part (a), most candidates knew how to rationalise the denominator and showed the appropriate multiplication by the fraction. Those who proceeded directly to the answer without showing the expansion of the brackets scored no marks. There were also a number of candidates who proceeded directly to $\frac{21\sqrt{3}-14\sqrt{5}}{7}$ which was able to score the first method mark for the denominator, however, no further marks could be scored as there was no method showing where the terms had come from, or any method shown of collecting terms. Some arithmetical errors occurred, but for the most part, candidates scored full marks.

In part (b), the majority of candidates knew to expand the brackets, group terms in x on one side, factorise and the divide by the bracket to isolate x. Neater solutions, using the "hence" option, then linked the algebraic fraction obtained in part (a) and gave an answer of five times that. Those who chose to rationalise the algebraic fraction obtained had most work to do and subsequently more arithmetical errors occurred.

Question 4

Many candidates were able to score at least five marks in this question on the intersection of a cubic function and a reciprocal function. However, very few scored full marks and, on the occasions where just a single mark was lost, it was usually in part (d) and, to a lesser extent, from incomplete factorisation in (b).

In part (a), most candidates correctly identified the asymptote $x = -2$ but this was sometimes given as coordinates or simply -2. Other common errors were $y = -2$ or $x = -0.5$.

In part (b), the majority of candidates successfully took out a linear factor, which was usually x but failed to complete the full process of the factorisation. Frequently $(x+2)^2$ followed the correct initial stage without the complete factorisation being shown. A significant number of candidates misunderstood what they were being asked in the question and attempted to solve the cubic, but they were not penalised for this unless they just tried to write down the solutions without factorising.

Of those who attempted the curve sketch in part (c), the majority correctly sketched a positive cubic, although the curvature was often dubious. Candidates lost marks by either having the cubic in the wrong location, from not identifying $x = -2$ as a repeated root and at the point where the asymptote was drawn, or, as the equation only has two roots, making the incorrect assumption that it was a quadratic function. The third B mark was often lost by candidates neglecting to mark the point $(-2, 0)$ on their diagram, even though it might have been mentioned in the working beforehand, or they may have plotted the point but did not have a curve going through this point. A few candidates neglected to draw graphs despite answers or relevant working seen in the other parts. There seemed a reluctance for candidates to improve their sketches using the copy of diagram 1 and sometimes they used Figure 1 instead.

Part (d) was either not attempted or typically unsuccessfully answered. Candidates often were unable to access the mark in part (d) because either their curve sketch was incorrect, or they would state the correct number of intersections but without a valid reason. Some attempted an algebraic justification rather than using their sketch which did not score.

Question 5

This question enabled candidates to demonstrate their knowledge and ability to apply of the cosine and sine rules. A wide range of mark traits was seen, with part (b) proving more challenging.

In part (a), almost all candidates were able to apply the cosine rule to write an equation in x and $\cos\theta$. The most successful approach was using the formula starting with $\cos\theta$ as its subject. There was relatively little scope for errors in its manipulation and both marks were usually gained. Occasionally candidates started from an incorrect formula and gained no marks. A few candidates lost the accuracy mark as they did not include "$\cos\theta =$" in any stage of their proof. The other widely used approach was using the formula as it appears in the formula booklet. Many candidates also scored full marks from this starting point, but errors in manipulation were often seen. Most commonly these were bracketing errors involving the expression for the coefficient of $\cos\theta$.

In part (b), the first mark was very accessible, and it was scored by the vast majority of candidates substituting $x = 2\sqrt{3}$ into the given expression for $\cos\theta$, then taking the arccos of the resulting value. There were many different methods then deployed to find the area of the overall triangle *ABC*. The most efficient method used by many candidates was to calculate the length *AB* using the sine rule, find the size of angle *ABC* by a simple subtraction and then use the area of a triangle formula $\frac{1}{2}ab\sin\theta$. Others opted to calculate angle *ABC* first, find *AC* using the sine rule, then once again applied the trigonometric formula for the triangle area. Both sets of these candidates were often successful in gaining full marks.

There were many very complex longer routes seen with the area formula attempted twice (for triangles *BCD* and *ABD*). These candidates mostly calculated all the missing lengths and angles in the given figure and often appeared to have no clear sense of direction. Some candidates only calculated the area of triangle *BCD* using their angle of θ and the lengths provided in the question. Their lack of any progress beyond this calculation meant that they only gained one of the five marks available as they had not demonstrated the problem-solving skills required to answer the question.

Only a few candidates who deployed a correct full method to find the total area avoided the use of $\frac{1}{2}ab\sin\theta$. They instead opted for use of $\frac{1}{2}\times \text{base} \times \text{perpendicular height}$ and this generally proved far less efficient. A small number of candidates lost accuracy marks due to premature rounding; some candidates would benefit from being reminded to retain a greater degree of accuracy when using any interim answers in further calculations.

Question 6

This was a challenging question, testing a candidate's ability at manipulating an equation involving more than one variable.

Part (a) was a "show that" question involving two variables, *x,* and *p*. The majority of candidates who attempted this were able to recognise that they needed to multiply both sides by $(x+p)$. Many candidates chose to multiply out the brackets on the left-hand side to achieve $4p-8x$ first; there were some responses that then left out the brackets (invisible brackets) or did not multiply the second term correctly, if at all. To achieve the first mark though, candidates were expected to collect terms on one side and several failed to do this, thus not scoring any marks. Some tried to form a three-term quadratic in *p* and made no further progress. There was much confusion on how to substitute the coefficients of the resulting quadratic into the discriminant, with many incorrect variations given for the three coefficients *a*, *b,* and *c*. For those candidates who had the correct combination, most were able to go on to form a correct inequality. Most candidates used the correct inequality sign >, given that there were two distinct roots. Those candidates who divided both sides of the inequality by 16 early on made the simplification much easier. Many candidates failed to gain the final A mark due to invisible brackets, or other errors seen in their working. There were many attempts that scored no marks for this part of the question and a few who did not attempt this at all.

In part (b), many candidates failed to notice or take heed of the warning at the top of the question regarding solutions that relied on the use of technology not being acceptable. This particular part also emphasised that the use of algebra was required. Many candidates used their calculators to find the critical values, not achieving the first mark at all and this also prevented the final mark from being scored, too. There were a few cases where the candidate did not state the quadratic formula, or did not use it correctly, losing the first mark. Most candidates used the quadratic formula or factorised to find their critical values. There was a very mixed rate of success at finding the correct range, with many opting for the inner region instead of the outer region for their critical values, thus not scoring the second method mark. Candidates who sketched the quadratic were more successful at identifying the correct regions. Some candidates were only able to achieve the second mark by selecting the outer region, despite not gaining the first mark. For those who did successfully find the correct region, many went on to score all three available marks. Some candidates used the word "and" which lost them the third mark or used *x* instead of *p*, which also lost the third mark. Very few candidates had the incorrect use of inequalities $4<p<-\frac{2}{3}$ but were able to achieve the second mark with this. The use of set notation was very rarely seen. Some gave "$p>4$ and $p>-\frac{2}{3}$" as their final answer which lost the final mark.

Question 7

This question tested a candidate's ability to use the first derivative of a function to find the equation of a normal to a curve at a particular point and to also find the equation of the curve.

Most candidates correctly attempted both (i) and (ii) and therefore achieved full marks. The most common errors were candidates proceeding to use the gradient of the curve and therefore finding the equation of the tangent to *C* at *P*, rather than the equation of the normal as required. When candidates did find the negative reciprocal of their gradient in (i), they were then successful in finding the equation of the normal. Some candidates did lose the accuracy mark due to a sign error in their final answer following poor rearranging of the equation. Others lost the accuracy mark by not meeting the demand of the question for *a*, *b,* and *c* to be integers and leaving their answer in an incorrect form.

Most candidates were able to confidently simplify $\mathrm{f}'(x)$ by splitting the fraction into three separate terms. A small number of candidates incorrectly achieved $\frac{5}{2}x^{\frac{1}{2}}$ as the middle term, missing the negative in the power, and a few candidates made an error simplifying the final term reaching $-\frac{7}{4}x$ instead of $-\frac{7}{4}$. A few candidates incorrectly multiplied the numerator by 4, while others added the numerator to the term of the denominator rather than dividing by it. However, the vast majority of candidates correctly integrated their expression. Some candidates made errors with the coefficients so, for example, rather than dividing by $\frac{5}{2}$, multiplying by it instead. A small number of candidates forgot to find the constant of integration. Those who remembered were mostly successful in finding the correct value for the constant, then stating their f(x) equation to finish the question, although quite a few made arithmetical slips. A small number failed to write out f(x) after finding *c* correctly, thus forfeiting the final mark.

Question 8

Overall, this question proved harder than anticipated for candidates to gain full marks as often even the more competent candidates did not heed the comment about the use of calculator technology, nor the instruction in part (b) to use algebra.

Part (a) involved eliminating *y* from two cartesian equations of curves resulting in the given quartic equation in *x*. Full marks were commonly awarded and most candidates gained at least one mark. A good number of candidates used the approach seen on the main mark scheme: substituting the second equation into the first, expanding the brackets, multiplying by 2 and rearranging to obtain the given result. Candidates who used this method usually gained both available marks. The majority of candidates however chose to make *y* the subject of the first equation and equate it to the second. This method was more complicated, and errors were often seen, most commonly occurring when multiplying through by $2x$. This often seemed to be a result of the layout of the solution and confusion over their own intent. In both methods there was often a reluctance to cancel out the two terms in $5x$ (or $10x$ or even just 5) until both were on the same side of the equation. While this did not often result in lost marks, it did result in more protracted solutions.

In part (b), most candidates appeared to recognise that the given quartic equation was in fact a hidden quadratic and solved for x^2 and then square rooted their positive result to find the x values. Despite recognising the need to do this, many of these candidates still used their calculators to solve the hidden quadratic which precluded them from both the first and the final mark. A few candidates simply forgot to square root the positive root of their quadratic, or only considered the positive square root resulting in only one value for x. Both these oversights meant that no further progress could be made. For those candidates with exactly two roots to the given quartic in the required form, it proved particularly challenging for many of them to obtain the corresponding y coordinates in exact form. Quite a few resorted to using decimals, despite the instruction in the question to find the exact distance of PQ. Another significant number assumed that the y values were both zero and the distance PQ was simply $|x_2 - x_1|$. For those candidates with x and y values in the required form, the use of the distance formula for finding length PQ was often not well executed, probably as a result of the irrational values involved. There were some very competent and efficient solutions gaining full marks, but these were uncommon.

Question 9

This question required an understanding of radians, and it was pleasing to see many candidates able to make progress with the question. However, candidates appeared to struggle with the problem-solving element to this question which resulted in slips in their working and incorrect methods overall.

In part (a), most candidates wrote $\frac{0.72}{0.6} = 1.2$ with confidence. Some writing $\frac{6}{5} = 1.2$ as confirmation, perhaps doing $\frac{0.72}{0.6}$ on their calculator to make sure. Almost all candidates scored this mark. Those who failed to score this mark almost always failed to score on further work and had no credible mathematics in their response. Some completed part (a) with no further work.

Both parts (b) and (c) required a plan and candidates who set this out first, achieved the best results. Most candidates scored the B1 with a correct expression for the large sector. Many chose to convert the 0.6 radians into degrees (or even as an expression perhaps thinking this was required as part of the solution) each time it was used throughout the question. This complicated the expressions and resulted in some errors. The most common error was not to include or deal correctly with the small sector. This could be subtracted from the large sector or added to the given area of 90; on occasions it was omitted entirely giving $\frac{1}{2} \times 0.6 \times (x + 1.2)^2 = 90$ and in some cases this was subtracted from 90.

In part (c), most candidates gave a correct value for x having solved the quadratic, with some as a decimal and some in surd form. Premature rounding of this answer to 16 was limited. The majority of candidates made a good attempt at the perimeter. A common error was omitting the 1.2 in the calculation to find the arc QR or, more prevalent, was omitting the arc PS. Some inevitably omitted the units and so lost an easy final mark having achieved a correct value. Many answers rounded to 16.1, however some of these were from incorrect methods and so the final two marks could not be scored.

Question 10

Many candidates were able to score at least two marks on this question, although it was very rare for candidates to score full marks.

In parts (a)(i) and (a)(ii), of the candidates that attempted this part of the question, the responses tended to be mixed with $n = 3$ being easier to obtain than 1080. A common misconception was to give an answer of $\frac{1}{3}$ i.e. an understanding of the transformation but incorrect application of this to the equation given. Some candidates found 1080 in radians or worked out it was three times greater but multiplied by 180 instead of 360.

In part (b), most candidates were able to obtain at least the first B mark due to either -3 or 1620 appearing. Occasionally 1620 was given on its own, but usually it was for seeing $"y" = -3$. A common incorrect value for *x* was 540. It was very rare to see these values written the wrong way around, although this was not penalised for the first mark.

Part (c) was either missed out or was poorly attempted, with very few candidates correctly solving for *k*. Candidates were able to use the coordinates given to form a pair of simultaneous equations in *a* and *k*, but often progressed no further, as they did not spot $2\sin(-a) = -2\sin(a)$. There were some attempts that used the numerical methods, but these were rarely successful.

Question 11

Overall, this was a well attempted question, considering it was the last one on the paper. Very few candidates left this entirely blank, with most at least attempting the first part.

Part (a) was an accessible part of the question, with the majority of candidates attempting and recognising that they needed to complete the square by removing a factor of 2 at the start. Many candidates chose to take out a factor of 2 from all three terms and this sometimes caused issues later on in their work, mostly from forgetting to multiply the constant term by 2 again and thus only achieving two out of the three marks available. However, many candidates were successful and were able to achieve full marks here. Some candidates were only able to achieve the first mark, failing to halve the second term after taking out a factor of 2. A small minority of candidates chose to expand the general completed square form and compare coefficients, often with some success.

The majority of candidates in part (b) were able to follow through their minimum point from their completed square form. There were a number of candidates who did not recognise that they could pick the minimum point from the completed square form and found it through differentiation instead. It is worthwhile for a candidate to note the number of marks available for a question part before embarking on longer methods.

Part (c) was well attempted with many candidates able to achieve the three marks available. Those who used the standard equation to find the gradient were generally the most successful here and even if their minimum point was incorrect, they were able to achieve the two method marks. Many candidates chose to form simultaneous equations in *m* and *c* using the equation of a straight line and the coordinates of the two points. This approach gave mixed results, although many were successful. There were a number of candidates who differentiated and found the gradient of the curve at the point $x = -1$, so they missed a key point and did not score any marks, even if they went on to use the equation of a straight line correctly. Some candidates thought the gradient could be found by substituting $x = -1$ into the equation for *C*.

Part (d) was generally well attempted by the majority of candidates. Of those who did attempt it, many were able to score the first two marks, some as a follow through on their answer to part (c). For those who attempted it and were not successful, this was due to either having the inequalities the wrong way around, or from using 0 or *R* instead of y. The third mark was much more difficult to achieve with many candidates forgetting to include $y > 0$. Some candidates went further and gave $0 < x < \frac{5}{2}$ or less frequently $0 < y < 20$ which was acceptable. In addition, some candidates gave only three inequalities instead of four. Use of strict inequalities (< and >) throughout was fairly common and inconsistent use of these with inclusive inequalities was fairly rare.